Practice problems for Conditional Probability Distributions.

Problem 1. The amount of kerosene, in thousands of liters, in a tank at the beginning of any day is a random amount Y from which a random amount X is sold during that day. Suppose that the tank is not resupplied during the day so that $x \leq y$, and assume that the joint density function of these variables is

$$f(x) = \begin{cases} 2, & 0 < x < y,\ 0 < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Determine if X and Y are independent.

(b) Find $P\left(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{3}{4}\right)$.

Problem 2. Three cards are drawn without replacement from the 12 face cards (jacks, queens and kings) of an ordinary deck of 52 playing cards. Let X be the number of kings selected and Y the number of jacks. Find

(a) the joint probability distribution of X and Y;

(b) $P\left[(X, Y) \in A\right]$, where A is the region given by $\{(x, y) | x + y \geq 2\}$.

Problem 3. Given the joint density function

$$f(x) = \begin{cases} \frac{x(1+3y^2)}{4}, & 0 < x < 2,\ 0 < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find $g(x)$, $h(y)$, $f(x|y)$, and evaluate $P\left(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{1}{3}\right)$.

Problem 4. Let X, Y, and Z have the joint probability density function

$$f(x, y, z) = \begin{cases} kxy^2z, & 0 < x < 1,\ 0 < y < 1, 0 < z < 2 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find k.

(b) Find $P\left(X < \frac{1}{4}, Y > \frac{1}{2}, 1 < Z < 2\right)$.

Problem 5. The joint probability density function of the random variables X, Y and Z is

$$f(x,y,z) = \begin{cases} \frac{4\ xyz^2}{9}, & 0 < x < 1,\ 0 < y < 1, 0 < z < 3 \\ 0, & \text{elsewhere.} \end{cases}$$

Find

(a) the joint marginal density function of Y and Z;

(b) the marginal density of Y;

(c) $P\left(\frac{1}{4} < X < \frac{1}{2}, Y > \frac{1}{3}, 1 < Z < 2\right)$;

(d) $P\left(0 < X < \frac{1}{2} | Y = \frac{1}{4}, Z = 2\right)$.