Practice problems for Expectation Value, Variance and Covariance.

Problem 1. Assume that two random variables (X, Y) are uniformly distributed on a circle with radius a. Then the joint probability density function is

$$f(x,y) = \begin{cases} \frac{1}{\pi a^2}, & x^2 + y^2 \leq a^2, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the expected value of X.

Problem 2. The probability distribution of X, the number of imperfections per 10 meters of synthetic fabric in continuous rolls of uniform width, is given as

x	0	1	2	3	4
f(x)	0.41	0.37	0.16	0.05	0.01

Find the average number of imperfections per 10 meters of this fabric.

Problem 3. By investing in a particular stock, a person can make a profit in one year of \$4000 with probability 0.3 or take a loss of \$1000 with probaiblity 0.7. What is this person's expected gain? What is the variance?

Problem 4. A private pilot wishes to insure his airplane for \$200,000. The insurance company estimates that a total loss may occur with probability 0.002, a 50% loss with probability 0.01, and a 25% loss with probability 0.1. Ignoring all other partial losses, what premium should the insurance company charge each year to realize an average profit of \$500?

Problem 5. Let X be a random variable with the following probability distribution

x	-3	6	9
f(x)	1/6	1/2	1/3

(a) Find $\mu_{g(x)}$ where $g(X, Y) = (2X + 1)^2$.

(b) Find the variance.

Problem 6. Suppose that X and Y have the following joint probability function

		x	
	f(x,y)	2	4
	1	0.10	0.15
y	3	0.20	0.30
	5	0.10	0.15

(a) Find the expected value of $g(X,Y) = XY^2$.

(b) Find μ_X, μ_Y.

Problem 7. The random variable X, representing the number of errors per 100 lines of software code, has the following probability distribution,

x	2	3	4	5	6
f(x)	0.01	0.25	0.40	0.30	0.04

(a) Find the variance of X.

(b) Find the mean and variance of the discrete random variable $Z = 3x - 2$.

Problem 8. A privately owned liquor store operates both a drive-in facility and a walk-in facility. On a randomly selected day, let X and Y, respectively, be the proportions of the time that the drive-in and walk-in facilities are in use, and suppose that the joint density function of these random variables is

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y), & 0 \le x \le 1 \text{ and } 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the covariance of X and Y.

Problem 9. If X and Y are independent random variables with variances $\sigma_X^2 = 5$ and $\sigma_Y^2 = 3$ find the variance of the random variable $Z = -2X + 4Y - 3$.

Problem 10. Suppose that X and Y are independent random variables with probability densities

$$g(x) = \begin{cases} \frac{8}{x^3}, & x > 2, \\ 0, & \text{elsewhere.} \end{cases}$$

and

$$h(y) = \begin{cases} 2/y, & 0 < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the expected value of $Z = XY$.