

Practice problems for functions of a random variable

Problem 1. Let X be a random variable with probability distribution

$$f(x) = \begin{cases} \frac{1}{3}, & x = 1, 2, 3 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability distribution of the random variable $Y = 2X - 1$.

Problem 2. The hospital period, in days, for patients following treatment for a certain type of kidney disorder is a random variable $Y = X + 4$, where X has the density function

$$f(x) = \begin{cases} \frac{32}{(x+4)^3}, & x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find the probability density function of the random variable Y .
- (b) Using the density function of Y , find the probability that the hospital period for a patient following this treatment will exceed 8 days.

Problem 3. The speed of a molecule in a uniform gas at equilibrium is a random variable V whose probability distribution is given by

$$f(v) = \begin{cases} kv^2e^{-bv^2}, & v > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

where k is an appropriate constant and b depends on the absolute temperature and mass of the molecule. Find the probability distribution of the kinetic energy of the molecule W , where $W = mV^2/2$.