

Practice problems for the Normal Distribution and the Central Limit Theorem

Problem 1. Given a standard normal distribution, find the area under the curve which lies

- (a) to the left of $z = 1.43$;
- (b) to the right of $z = -0.89$;
- (c) between $z = -2.16$ and $z = -0.65$;
- (d) to the left of $z = -1.39$;
- (e) to the right of $z = 1.96$;
- (f) between $z = -0.48$ and $z = 1.74$;

Problem 2. Given a standard normal distribution, find the value of k such that

- (a) $P(Z < k) = 0.0427$;
- (b) $P(Z > k) = 0.2946$;
- (c) $P(-0.93 < Z < k) = 0.2946$.

Problem 3. Given the normally distributed variable X with mean 18 and standard deviation 2.5, find

- (a) $P(X < 15)$;
- (b) the value of k such that $P(Z < k) = 0.2236$;
- (c) the value of k such that $P(Z > k) = 0.1814$;
- (d) $P(17 < X < 21)$.

Problem 4. A soft-drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 milliliters,

- (a) what fraction of the cups will contain more than 224 milliliters?
- (b) what is the probability that a cup contains between 191 and 209 milliliters?
- (c) how many cups will probably overflow if 230 milliliter cups are used for the next 1000 drinks?
- (d) below what value do we get the smallest 25% of the drinks?

Problem 5. The random variable X , representing the number of cherries in a cherry puff, has the following probability distribution

x	4	5	6	7
$P(X=x)$	0.2	0.4	0.3	0.1

- (a) Find the mean μ and the variance σ^2 of X .
- (b) Find the mean $\mu_{\bar{x}}$ and the variance $\sigma_{\bar{x}}^2$ of the mean \bar{X} for random samples of 36 cherry puffs.
- (c) Find the probability that the average number of cherries in 36 cherry puffs will be less than 5.5.

Problem 6. The average life of a bread-making machine is 7 years, with a standard deviation of 1 year. Assuming that the lives of these machines follow approximately a normal distribution, find

- (a) the probability that the mean life of a random sample of 9 such machines falls between 6.4 and 7.2 years;
- (b) the value of x to the right of which 15% of the means computed from random samples of size 9 would fall.