Solutions to practice problems for Conditional Distributions

Problem 1.

(a) They are dependent (X is always smaller than Y).

(b) $h(y) = \int_0^y 2dx = 2y$
$f(x|y) = \frac{f(x,y)}{h(y)} = \frac{2}{2y} = \frac{1}{y}$
$P\left(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{3}{4}\right) = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{\frac{3}{4}} = \frac{1}{3}$.

Problem 2.

(a)

y, x	0	1	2	3
0	$\frac{1}{55}$	$\frac{6}{55}$	$\frac{6}{55}$	$\frac{1}{55}$
1	$\frac{6}{55}$	$\frac{16}{55}$	$\frac{6}{55}$	
2	$\frac{6}{55}$	$\frac{6}{55}$		
3	$\frac{1}{55}$			

(b) A = {(x,y): (1,1);(2,0);(0,2);(1,2);(2,1);(3,0);(0,3)}
$P\left[(X,Y) \in A\right] = \frac{16}{55} + \frac{6}{55} + \frac{6}{55} + \frac{6}{55} + \frac{6}{55} + \frac{1}{55} + \frac{1}{55} = \frac{42}{55}$.

Problem 3.
$g(x) = \int_\infty^\infty f(x,y)\, dy = \int_0^1 \frac{x(1+3y^2)}{4}\, dy = \frac{x}{2}$,

$h(y) = \int_\infty^\infty f(x,y)\, dx = \int_0^2 \frac{x(1+3y^2)}{4}\, dx = \frac{1+3y^2}{2}$,

$f(x|y) = \frac{f(x,y)}{h(y)} = \frac{x(1+3y^2)/4}{1+3y^2)/2} = \frac{x}{2}$,

$P\left(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{1}{3}\right) = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{x}{2} dx = \frac{3}{64}$.

Problem 4.

(a) $\int_0^2 \int_0^1 \int_0^1 k\ xy^2z\ dx\ dy\ dz = 1 \Leftrightarrow$
$k = \left[\int_0^2 \int_0^1 \int_0^1 xy^2z\ dx\ dy\ dz\right]^{-1} \Leftrightarrow$
$k = 3$.

(b) $P\left(X < \frac{1}{4}, Y > \frac{1}{2}, 1 < Z < 2\right) = \int_1^2 \int_{\frac{1}{2}}^1 \int_0^{\frac{1}{4}} 3\ xy^2z\ dx\ dy\ dz = \frac{21}{512}$.

Problem 5.

(a) $f(x,y) = \int_0^1 \frac{4}{9}\ xyz^2\ dx = \frac{2}{9}yz^2$

(b) $h(y) = \int_0^3 \frac{2}{9}yz^2\ dz = 2y$

(c) $P\left(\frac{1}{4} < X < \frac{1}{2}, Y > \frac{1}{3}, 1 < Z < 2\right) = \int_1^2 \int_{\frac{1}{3}}^1 \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{4}{9}\ xyz^2\ dx\ dy\ dz = \frac{7}{162}$;

(d) $fx|y,z = \frac{f(x,y,z)}{f(y,z)} = \frac{2/9xyz^2}{\frac{2}{9}yz^2} = 2x$
$P\left(0 < X < \frac{1}{2} | Y = \frac{1}{4}, Z = 2\right) = \int_0^{\frac{1}{2}} 2x\ dx = \frac{1}{4}$.